CBSE 2012 DELHI SUPPLEMENTARY EXAMINATION
[Solutions With Detailed Explanations]

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A
(Question numbers 01 to 10 carry 1 mark each.)

Q01. The value of the determinant of a matrix A of order 3×3 is 4. Find the value of |5A|.

Sol. As $|kA| = k^n|A|$, where n is the order of matrix A.
So, $|5A| = 5^3|A| = 125(4) = 500$.

Q02. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

Sol. We have $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ $\Rightarrow 3A = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + B$ $\Rightarrow 3A = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow A = \frac{1}{3}\begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix}$ $\therefore\ A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$.

Q03. For a 2×2 matrix $A = \left[a_{ij}\right]$, whose elements are given by $a_{ij} = \frac{(i+2j)^2}{4}$, write the value of a_{21}.

Sol. As $a_{ij} = \frac{(i+2j)^2}{4}$. So for a_{21}, put $i = 2,\ j = 1$ to get $a_{21} = \frac{[2+2(1)]^2}{4} = 4$.
So, the value of a_{21} is 4.

Q04. If $f : R \to R$ defined as $f(x) = \frac{2x-7}{4}$ is an invertible function, write $f^{-1}(x)$.

Sol. Let $f(x) = y = \frac{2x-7}{4}$ $\Rightarrow 4y = 2x - 7$ $\Rightarrow x = \frac{4y+7}{2}$

$\Rightarrow f^{-1} = \frac{4y+7}{2}$ Hence $f^{-1}(x) = \frac{4x+7}{2}$.

Q05. If the equation of a line AB are $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$ then, determine the direction cosines of a line parallel to AB.

Sol. Equation of line AB is: $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6} \Rightarrow \frac{x-3}{3} = \frac{y-(-2)}{-2} = \frac{z-(-2)}{6}$.
So, direction ratios of line AB are 3,–2, 6.
$\therefore$ Direction cosines of line AB are: $\pm\frac{3}{\sqrt{(3)^2+(-2)^2+(6)^2}}, \pm\frac{(-2)}{\sqrt{49}}, \pm\frac{6}{7}$ i.e., $\pm\frac{3}{7}, \mp\frac{2}{7}, \pm\frac{6}{7}$.

Since parallel lines have same d.c.'s so, d.c.'s of a line parallel to AB are: $\pm\frac{3}{7}, \mp\frac{2}{7}, \pm\frac{6}{7}$.

Q06. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$, where $xy < 1$, find the value of $x + y + xy$.

Sol. We have $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ $\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{x+y}{1-xy} = \tan\frac{\pi}{4}$ $\Rightarrow \frac{x+y}{1-xy} = 1$ $\Rightarrow x + y = 1 - xy$ $\Rightarrow x + y + xy = 1$

So, value of $x + y + xy$ is 1.

Q07. Find a unit vector parallel to the sum of the vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 5\hat{k}$.

Sol. Let $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + (2\hat{i} - 3\hat{j} + 5\hat{k}) = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

We have $|\vec{r}| = \sqrt{(3)^2 + (-2)^2 + (6)^2} = 7$. $\qquad \therefore \ \hat{r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$.

Hence the required unit vector is $\dfrac{3}{7}\hat{i} - \dfrac{2}{7}\hat{j} + \dfrac{6}{7}\hat{k}$.

Q08. Write the value of λ for which the vectors $\hat{i} + 2\lambda\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ are perpendicular.

Sol. If the given vectors are perpendicular then, $(\hat{i} + 2\lambda\hat{j} + \hat{k}).(2\hat{i} + \hat{j} - 3\hat{k}) = 0$

i.e., $2 + 2\lambda - 3 = 0$ $\qquad \Rightarrow \lambda = \dfrac{1}{2}$. $\qquad$ So, value of λ is $\dfrac{1}{2}$.

Q09. Write the value of $\displaystyle\int_0^1 \frac{e^x}{1 + e^{2x}} dx$.

Sol. Let $\mathrm{I} = \displaystyle\int_0^1 \frac{e^x}{1 + e^{2x}} dx$ $\qquad \Rightarrow \mathrm{I} = \displaystyle\int_0^1 \frac{e^x}{1 + (e^x)^2} dx$

Put $e^x = t \Rightarrow e^x dx = dt$. $\qquad$ Also, when $x = 0 \Rightarrow t = e^0 = 1$. And when, $x = 1 \Rightarrow t = e$.

$\therefore \ \mathrm{I} = \displaystyle\int_1^e \frac{dt}{1 + t^2}$ $\qquad \Rightarrow \mathrm{I} = \left[\tan^{-1}(t)\right]_1^e = \tan^{-1}(e) - \tan^{-1}(1)$

Hence $\displaystyle\int_0^1 \frac{e^x}{1 + e^{2x}} dx = \tan^{-1}(e) - \frac{\pi}{4}$.

Q10. Write the value of $\displaystyle\int \frac{\sec^2 x}{\mathrm{cosec}^2 x} dx$.

Sol. Let $\mathrm{I} = \displaystyle\int \frac{\sec^2 x}{\mathrm{cosec}^2 x} dx$ $\qquad \Rightarrow \mathrm{I} = \displaystyle\int \frac{1/\cos^2 x}{1/\sin^2 x} dx = \int \tan^2 x \, dx$

$\Rightarrow \mathrm{I} = \displaystyle\int (\sec^2 x - 1) dx$ $\qquad \therefore \ \mathrm{I} = \tan x - x + k$, where *k* is integral constant.

SECTION – B
(Question numbers 11 to 22 carry 4 marks each.)

Q11. By computing the shortest distance between the following pair of lines, determine whether they intersect or not: $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$; $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$?

Sol. We have $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ $\qquad \Rightarrow \vec{a}_1 = \hat{i} - \hat{j}, \ \vec{b}_1 = 2\hat{i} + \hat{k}$.

And, $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$ $\qquad \Rightarrow \vec{a}_2 = 2\hat{i} - \hat{j}, \ \vec{b}_2 = \hat{i} - \hat{j} - \hat{k}$.

So, $\vec{a}_2 - \vec{a}_1 = \hat{i}$,

And, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \hat{i} + 3\hat{j} - 2\hat{k}$.

Now shortest distance, S.D. $= \dfrac{\left|(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$ *units*

$\Rightarrow \qquad = \dfrac{\left|(\hat{i} + 3\hat{j} - 2\hat{k}).(\hat{i})\right|}{\sqrt{(1)^2 + (3)^2 + (-2)^2}}$ *units* $= \dfrac{1}{\sqrt{14}}$ *units* $\qquad \therefore \ \text{S.D.} = \dfrac{\sqrt{14}}{14}$ *units*

As $\text{S.D.} \neq 0$, so the given two lines do not intersect as for intersecting lines, S.D. comes out to be zero.

Q12. An urn contains 4 white and 6 red balls. Four balls are drawn at random (without replacement) from the urn. Find the probability distribution of the number of white balls.

Sol. Let X be the number of white balls drawn from the urn. So, X can take values 0, 1, 2, 3, 4.
Number of white balls in the urn = 4, Number of white balls in the urn = 6.

Total number of balls in the urn = 4 + 6 = 10.
The probability distribution of the number of white balls drawn from the urn is given in the table below:

X	0	1	2	3	4
P(X)	$\frac{^6C_4}{^{10}C_4}=\frac{15}{210}$	$\frac{^4C_1\,^6C_3}{^{10}C_4}=\frac{80}{210}$	$\frac{^4C_2\,^6C_2}{^{10}C_4}=\frac{90}{210}$	$\frac{^4C_3\,^6C_1}{^{10}C_4}=\frac{24}{210}$	$\frac{^4C_4}{^{10}C_4}=\frac{1}{210}$

Q13. Find the intervals in which the function given by $f(x)=\sin x+\cos x,\ 0\le x\le 2\pi$ is
(a) increasing, (b) decreasing.

OR

For the curve $y=4x^3-2x^5$, find all those points at which the tangent passes through the origin.

Sol. We have $f(x)=\sin x+\cos x$

Differentiating with respect to x both sides: $f'(x)=\cos x-\sin x$.

For $f'(x)=0$, $\cos x-\sin x=0 \quad \Rightarrow \tan x=1$

$\therefore\ x=\frac{\pi}{4},\frac{5\pi}{4}\in[0,2\pi] \qquad \therefore\ x=\frac{\pi}{4},\frac{5\pi}{4}\in[0,2\pi]$

Interval	Sign of $f'(x)$	Nature of $f(x)$
$\left(0,\frac{\pi}{4}\right)$	Positive	Increasing
$\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$	Negative	Decreasing
$\left(\frac{5\pi}{4},2\pi\right)$	Positive	Increasing

So, $f(x)$ is increasing on $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right)$. And, $f(x)$ is decreasing on $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$.

OR

We have: $y=4x^3-2x^5$ …(i)

Let $P(x_1,y_1)$ be the required point on the given curve. So, $y_1=4x_1^3-2x_1^5$ …(ii)

Differentiating (i) w.r.t. x both sides, we have: $\frac{dy}{dx}=12x^2-10x^4$

$$\therefore\ \left.\frac{dy}{dx}\right]_{at\ (x_1,y_1)}=12x_1^2-10x_1^4=m_T$$

The equation of tangent at $P(x_1,y_1)$ is: $y-y_1=(12x_1^2-10x_1^4)(x-x_1)$

As the tangents at $P(x_1,y_1)$ are passes through the origin so, we have:

$0-y_1=(12x_1^2-10x_1^4)(0-x_1) \qquad \Rightarrow y_1=12x_1^3-10x_1^5$ ...(iii)

By (ii) and (iii), we have: $4x_1^3-2x_1^5=12x_1^3-10x_1^5 \qquad \Rightarrow x_1^3(x_1^2-1)=0$

$\therefore\ x_1=-1,0,1 \qquad \Rightarrow y_1=-2,0,2$.

So, the required points on the given curves at which the tangents passes through the origin are (0, 0), (1, 2) and (–1,–2).

Q14. Evaluate: $\int\left(\frac{1+\sin x}{1+\cos x}\right)e^x dx$. **OR** Evaluate: $\int\frac{x^2}{(x\sin x+\cos x)^2}dx$.

Sol. Let $I=\int\left(\frac{1+\sin x}{1+\cos x}\right)e^x dx \qquad \Rightarrow I=\int\left(\frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right)e^x dx$

$\Rightarrow \text{I} = \int \left(\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2} \right) e^x dx \qquad \Rightarrow \text{I} = \int e^x \tan\frac{x}{2}\, dx + \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} \right) dx$

Applying by parts in the first integral, we have:

$\text{I} = \tan\frac{x}{2} \int e^x\, dx - \int \left[\frac{d}{dx}\left(\tan\frac{x}{2} \right) \int e^x\, dx \right] dx + \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} \right) dx$

$\Rightarrow \text{I} = e^x \tan\frac{x}{2} - \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} \right) dx + \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} \right) dx \qquad \therefore \ \text{I} = e^x \tan\frac{x}{2} + k.$

OR Let $\text{I} = \int \frac{x^2}{(x\sin x + \cos x)^2} dx$

$\Rightarrow \text{I} = \int (x\sec x)\, \frac{x\cos x}{(x\sin x + \cos x)^2} dx$ [Applying integral by parts

$\Rightarrow \text{I} = (x\sec x) \int \frac{x\cos x}{(x\sin x + \cos x)^2} dx - \int \left[\frac{d}{dx}(x\sec x) \int \frac{x\cos x}{(x\sin x + \cos x)^2} dx \right] dx$

Put $x\sin x + \cos x = t \Rightarrow x\cos x\, dx = dt$.

$\Rightarrow \text{I} = (x\sec x) \int \frac{dt}{(t)^2} - \int \left[(x\sec x\tan x + \sec x) \int \frac{dt}{(t)^2} \right] dx$

$\Rightarrow \text{I} = -\frac{(x\sec x)}{t} + \int \left[\frac{\sec x(x\tan x + 1)}{t} \right] dx$

$\Rightarrow \text{I} = -\frac{(x\sec x)}{x\sin x + \cos x} + \int \frac{\sec^2 x(x\sin x + \cos x)}{x\sin x + \cos x} dx$

$\Rightarrow \text{I} = -\frac{(x\sec x)}{x\sin x + \cos x} + \int \sec^2 x\, dx \qquad \Rightarrow \text{I} = -\frac{(x\sec x)}{x\sin x + \cos x} + \tan x + k$

$\Rightarrow \text{I} = \frac{\sin x}{\cos x} - \frac{x}{\cos x(x\sin x + \cos x)} + k \qquad \Rightarrow \text{I} = \frac{1}{\cos x}\left[\sin x - \frac{x}{(x\sin x + \cos x)} \right] + k$

$\therefore \ \text{I} = \frac{\sin x - x\cos x}{x\sin x + \cos x} + k.$

Q15. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, find $\frac{dy}{dx}$.

OR If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$.

Sol. Let $y = u + v$ where $u = x^{\sin x - \cos x}$ and $v = \frac{x^2 - 1}{x^2 + 1}$.

On differentiating w.r.t. x both sides, we have: $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(i)

Now, $u = x^{\sin x - \cos x}$

$\Rightarrow \log u = \log x^{\sin x - \cos x}$ [Taking logarithm on both the sides

$\Rightarrow \log u = (\sin x - \cos x)\log x$

$\frac{1}{u}\frac{du}{dx} = (\sin x - \cos x)\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\sin x - \cos x)$ [Differentiating w.r.t. x both the sides

$\therefore \ \frac{du}{dx} = x^{\sin x - \cos x}\left[\frac{\sin x - \cos x}{x} + (\cos x + \sin x)\log x \right]$ …(ii)

And, $v = \frac{x^2 - 1}{x^2 + 1} \qquad v = \frac{(x^2 + 1) - 2}{x^2 + 1} = 1 - \frac{2}{x^2 + 1}$

$$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}\left(1 - \frac{2}{x^2+1}\right) = 0 - 2\left(-\frac{1}{(x^2+1)^2}\right)(2x)$$

$$\therefore \frac{dv}{dx} = \frac{4x}{(x^2+1)^2} \qquad \ldots\text{(iii)}$$

So by (i), (ii) and (iii), we have:

$$\frac{dy}{dx} = x^{\sin x - \cos x}\left[\frac{\sin x - \cos x}{x} + (\cos x + \sin x)\log x\right] + \frac{4x}{(x^2+1)^2}.$$

OR

We have $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$

Differentiating x and y both with respect to t both the sides, we have:

$$\frac{dx}{dt} = \frac{d}{dt}\big(a(\cos t + t\sin t)\big)$$

$$\Rightarrow \frac{dx}{dt} = a\left(-\sin t + t\cos t + \sin t\right)$$

$$\Rightarrow \frac{dx}{dt} = at\cos t \qquad \ldots\text{(i)}$$

$$\frac{dy}{dt} = \frac{d}{dt}\big(a(\sin t - t\cos t)\big)$$

$$\frac{dy}{dt} = a\left(\cos t + t\sin t - \cos t\right)$$

$$\Rightarrow \frac{dy}{dt} = at\sin t \qquad \ldots\text{(ii)}$$

By (i) and (ii), we have:

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \left(at\sin t\right)\left(\frac{1}{at\cos t}\right) = \tan t$$

Differentiating with respect to x both sides, we have: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t) = \sec^2 t\ \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \sec^2 t\left(\frac{1}{at\cos t}\right) \qquad \Rightarrow \frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}.$$

Also, differentiating (ii) with respect to t both sides, we have: $\frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}(at\sin t)$

$$\Rightarrow \frac{d^2y}{dt^2} = a(t\cos t + \sin t).$$

Q16. Solve the following differential equation:

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x.$$

Sol. We have $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right) \qquad \ldots\text{(i)} \qquad \text{[Dividing both the sides by } x\cos\left(\frac{y}{x}\right)$$

Put $y = vx$.

On differentiating w.r.t. x both the sides, we get: $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Substituting the value of $\frac{dy}{dx}$ in (i), we have: $v + x\frac{dv}{dx} = \frac{vx}{x} + \sec\left(\frac{vx}{x}\right)$

$$v + x\frac{dv}{dx} = v + \sec v \qquad \Rightarrow x\frac{dv}{dx} = \sec v$$

$$\Rightarrow \int \cos v\, dv = \int \frac{dx}{x} \qquad \Rightarrow \sin v = \log|x| + k$$

$\therefore\ \sin\left(y/x\right) = \log|x| + k$ is the required solution of the given differential equation.

Q17. Let $f: \mathrm{N} \to \mathrm{N}$ be defined as $f(n) = \begin{cases} (n+1)/2, \text{ when } n \text{ is odd} \\ n/2, \text{ when } n \text{ is even} \end{cases}$ for all $n \in \mathrm{N}$.

State whether the function f is bijective. Justify your answer.

Sol. We have $f: \mathrm{N} \to \mathrm{N}$ defined as $f(n) = \begin{cases} (n+1)/2, \text{ when } n \text{ is odd} \\ n/2, \text{ when } n \text{ is even} \end{cases}$ for all $n \in \mathrm{N}$.

It can be easily observed that, $f(1) = \frac{1+1}{2} = 1$ and $f(2) = \frac{2}{2} = 1$ [By definition of *f*

$\therefore \; f(1) = f(2)$ where $1 \neq 2$.

Therefore, *f* is not one-one.

Consider a natural number *n* in co-domain N.

Case I: When *n* is odd.

There exists $n = 2r + 1$ for some $r \in \mathrm{N}$.

Then, there exists $4r+1 \in \mathrm{N}$ such that $f(4r+1) = \frac{4r+1+1}{2} = 2r+1$.

Case II: When *n* is even.

There exists $n = 2r$ for some $r \in \mathrm{N}$.

Then, there exists $4r \in \mathrm{N}$ such that $f(4r) = 4r/2 = 2r$.

Therefore, *f* is onto.

But since *f* is not one-one, so it is not bijective (as a bijective function is necessarily both one-one as well as onto function).

Q18. If the sum of two unit vectors $\hat{a}$ and $\hat{b}$ is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Sol. We have $|\hat{a}| = |\hat{b}| = |\hat{a} + \hat{b}| = 1$.

Now, $|\hat{a} + \hat{b}|^2 = (\hat{a} + \hat{b}).(\hat{a} + \hat{b})$

$= |\hat{a}|^2 + 2\hat{a}.\hat{b} + |\hat{b}|^2 \quad \Rightarrow (1)^2 = (1)^2 + 2\hat{a}.\hat{b} + (1)^2 \quad \Rightarrow 2\hat{a}.\hat{b} = -1$ ...(i)

Also, $|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b}).(\hat{a} - \hat{b})$

$= |\hat{a}|^2 - 2\hat{a}.\hat{b} + |\hat{b}|^2 \quad \Rightarrow |\hat{a} - \hat{b}|^2 = (1)^2 - (-1) + (1)^2$ [By using (i)

$\therefore \; |\hat{a} - \hat{b}| = \sqrt{3}$. [Hence Proved.

Q19. Using properties of determinants, prove the following:

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma).$$

Sol. **LHS:** Let $\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$ [Applying $R_1 \to R_1 + R_3$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & 1 & 1 \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$ [Taking $(\alpha+\beta+\gamma)$ common from R_1

$$=(\alpha+\beta+\gamma)\begin{vmatrix}0 & 0 & 1\\ \alpha^2-\beta^2 & \beta^2-\gamma^2 & \gamma^2\\ \beta-\alpha & \gamma-\beta & \alpha+\beta\end{vmatrix}$$ [Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$=(\alpha-\beta)(\beta-\gamma)(\alpha+\beta+\gamma)\begin{vmatrix}0 & 0 & 1\\ \alpha+\beta & \beta+\gamma & \gamma^2\\ -1 & -1 & \alpha+\beta\end{vmatrix}$$

Taking $(\alpha-\beta)$ and $(\beta-\gamma)$ common from C_1 and C_2 respectively.

$=(\alpha-\beta)(\beta-\gamma)(\alpha+\beta+\gamma)\left((-\alpha-\beta)-(-\beta-\gamma)\right)$ [Expanding along R_1

$=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

$=$ **RHS**. [Hence Proved.

Q20. Find the value of k so that the following function is continuous at $x=2$:

$$f(x)=\begin{cases}\dfrac{x^3+x^2-16x+20}{(x-2)^2}; & x\neq 2\\ \text{k}; & x=2\end{cases}.$$

Sol. As $f(x)$ is continuous at $x=2$, so $\text{LHL}(at\ x=2)=\text{RHL}(at\ x=2)=f(2)$ ...(i)

Now, **RHL at $x=2$:**

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}\frac{x^3+x^2-16x+20}{(x-2)^2}$$

$$=\lim_{x\to 2^+}\frac{(x-2)^2(x+5)}{(x-2)^2}=\lim_{x\to 2^+}(x+5)$$

$=2+5=7$.

Also, $f(2)=k$.

By (i), $k=7$.

Q21. Find the particular solution of the following differential equation, given that $x=2,\ y=1$:

$$x\frac{dy}{dx}+2y=x^2,\ (x\neq 0).$$

Sol. We have $x\dfrac{dy}{dx}+2y=x^2$ $\Rightarrow \dfrac{dy}{dx}+\left(\dfrac{2}{x}\right)y=x$

This is linear differential equation of the form $\dfrac{dy}{dx}+\text{P}(x)y=\text{Q}(x)$ where, $\text{P}(x)=\dfrac{2}{x},\ \text{Q}(x)=x$.

Now Integrating Factor, I.F. $=e^{\int \text{P}(x)dx}=e^{\int \frac{2}{x}dx}$

$\Rightarrow$ $\text{I.F.}=e^{2\log x}=e^{\log x^2}=x^2$.

So the solution of the given differential equation is given by:

$y(\text{I.F.})=\int(\text{I.F.})\text{Q}(x)\,dx$ $\Rightarrow$ $y(x^2)=\int(x^2)(x)\,dx$

$\Rightarrow$ $y(x^2)=\int x^3dx$ $\Rightarrow$ $y(x^2)=\dfrac{x^4}{4}+k$.

Since it is given that $x=2,\ y=1$ so, we have: $(1)(2^2)=\dfrac{2^4}{4}+k$ $\Rightarrow k=0$.

Hence, the particular solution of the given differential equation is: $x^2=4y$.

Q22. Prove that: $\tan^{-1}(1/2)+\tan^{-1}(1/5)+\tan^{-1}(1/8)=\pi/4$.

OR Solve for x: $\tan^{-1}\dfrac{x-1}{x-2}+\tan^{-1}\dfrac{x+1}{x+2}=\dfrac{\pi}{4}$.

Sol. **LHS:** $\tan^{-1}\dfrac{1}{2}+\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{8}$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}.\frac{1}{5}}\right)+\tan^{-1}\frac{1}{8}$$ [Using $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$= \tan^{-1}\frac{7}{9}+\tan^{-1}\frac{1}{8}=\tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}.\frac{1}{8}}\right) \qquad = \tan^{-1}\left(\frac{65}{65}\right)=\tan^{-1}(1)$$

$= \frac{\pi}{4} =$ **RHS**. [Hence Proved.

OR We have: $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right)=\frac{\pi}{4}$$ [Using $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$\Rightarrow \frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{x^2+x-2+x^2-x-2}{-3}=1 \qquad \Rightarrow 2x^2=1 \qquad \therefore\ x=\pm\frac{1}{\sqrt{2}}.$$

Both these values of x satisfy the given equation, so its solutions are $\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}$.

SECTION – C
(Question numbers 23 to 29 carry 6 marks each.)

Q23. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be hearts. Find the probability of the missing card to be a heart.

Sol. Let H, S, C and D denotes the following events:
H: The missing card is a Heart, S: The missing card is a Spade, C: The missing card is a Club,
D: The missing card is a Diamond.
Also let E denotes the event of 'drawing two Heart cards from the remaining cards'.
So, P(H) = 13/52, P(S) = 13/52, P(C) = 13/52, P(D) = 13/52.
Also $P(E|H)={}^{12}C_2/{}^{51}C_2$, $P(E|S)=P(E|C)=P(E|D)={}^{13}C_2/{}^{51}C_2$.
Now by using Bayes' Theorem, we have:

$$P(H|E)=\frac{P(H)P(E|H)}{P(H)P(E|H)+P(S)P(E|S)+P(C)P(E|C)+P(D)P(E|D)}$$

$$\Rightarrow\ =\frac{(13/52)\left({}^{12}C_2/{}^{51}C_2\right)}{(13/52)\left({}^{12}C_2/{}^{51}C_2\right)+(13/52)\left({}^{13}C_2/{}^{51}C_2\right)+(13/52)\left({}^{13}C_2/{}^{51}C_2\right)+(13/52)\left({}^{13}C_2/{}^{51}C_2\right)}$$

$$\Rightarrow\ =\frac{(13/52)\left({}^{12}C_2/{}^{51}C_2\right)}{(13/52)\left[\left({}^{12}C_2/{}^{51}C_2\right)+3\left({}^{13}C_2/{}^{51}C_2\right)\right]}$$

$$\Rightarrow\ =\frac{{}^{12}C_2/{}^{51}C_2}{{}^{12}C_2/{}^{51}C_2+3({}^{13}C_2/{}^{51}C_2)}=\frac{{}^{12}C_2/{}^{51}C_2}{[{}^{12}C_2+3({}^{13}C_2)]/{}^{51}C_2}=\frac{{}^{12}C_2}{{}^{12}C_2+3({}^{13}C_2)}$$

$$\Rightarrow\ =\frac{12\times11/2}{(12\times11/2)+3\times(13\times12/2)}=\frac{11}{11+39} \qquad \therefore\ P(H|E)=\frac{11}{50}.$$

Q24. Find the equation of the plane passing through the point (3,–3, 1) and perpendicular to the line joining the points (3, 4,–1) and (2,–1, 5). Also find the coordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane.

OR Find the distance of the point (3, 4, 5) from the plane $x+y+z=2$ measured parallel to the line $2x=y=z$.

Sol. The direction ratios of line joining the points (3, 4,–1) and (2,–1, 5) are –1, –5, 6.
Since this line is perpendicular to the required plane so, the normal vector of the required plane is given by, $\vec{m}=-\hat{i}-5\hat{j}+6\hat{k}$.

Given point on the plane is *say* A(3,–3, 1) so, $\overrightarrow{OA}=\vec{a}=3\hat{i}-3\hat{j}+\hat{k}$.

$\therefore$ Equation of plane is: $\vec{r}.\vec{m}=\vec{a}.\vec{m}$

$$\Rightarrow \quad \vec{r}.(-\hat{i}-5\hat{j}+6\hat{k})=(3\hat{i}-3\hat{j}+\hat{k}).(-\hat{i}-5\hat{j}+6\hat{k})$$

$$\Rightarrow \quad \vec{r}.(-\hat{i}-5\hat{j}+6\hat{k})=18 \quad \Rightarrow \quad \vec{r}.(\hat{i}+5\hat{j}-6\hat{k})+18=0$$

or, $x+5y-6z+18=0$ is the required equation of plane.

Let M be the foot of perpendicular on the plane drawn from the origin O(0, 0, 0).

So, equation of line OM: $\frac{x-0}{-1}=\frac{y-0}{-5}=\frac{z-0}{6}=\lambda$ (*say*)

Coordinates of any random point on this line OM is M($-\lambda,-5\lambda,6\lambda$).

Since M lies on the plane $x+5y-6z+18=0$ so, $-\lambda+5(-5\lambda)-6(6\lambda)+18=0 \quad \Rightarrow \lambda=9/31$.

Substituting the value of λ in M($-\lambda,-5\lambda,6\lambda$), we have: $\text{M}\left(-9/31,-45/31,54/31\right)$.

So, the coordinates of **foot of perpendicular** is: $\text{M}\left(-9/31,-45/31,54/31\right)$.

Equation of perpendicular line OM is: $\frac{x}{-1}=\frac{y}{-5}=\frac{z}{6}$.

Also, **length of perpendicular** drawn from origin O(0, 0, 0) to the plane $x+5y-6z+18=0$ is:

$$\frac{|0+5(0)-6(0)+18|}{\sqrt{(1)^2+(5)^2+(-6)^2}}=\frac{18}{\sqrt{62}}\ units.$$

OR

Let P(3, 4, 5).

The direction ratios of line $2x=y=z$ i.e., $\frac{x-0}{1/2}=\frac{y-0}{1}=\frac{z-0}{1}$ are $\frac{1}{2},1,1$ or $1,2,2$.

Equation of line through P and parallel to the line $2x=y=z$ is:

$$\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda \text{ (say)} \qquad \ldots\text{(i)}$$

The coordinates of any random point on (i) is, $\text{Q}(\lambda+3,2\lambda+4,2\lambda+5)$.

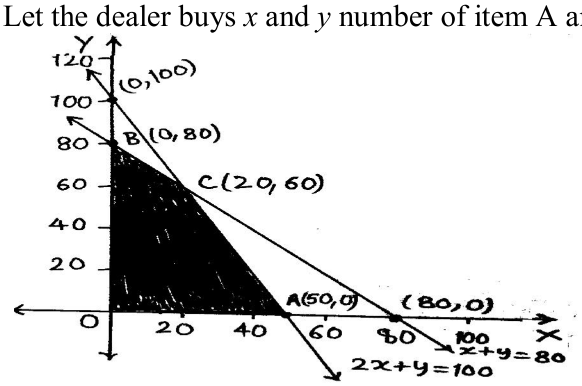
If Q lies on the plane $x+y+z=2$ then, $\lambda+3+2\lambda+4+2\lambda+5=2 \Rightarrow \lambda=-2$.

Substituting the value of $\lambda=-2$ in $\text{Q}(\lambda+3,2\lambda+4,2\lambda+5)$, we get $\text{Q}(1,0,1)$.

So, required distance $\text{PQ}=\sqrt{(1-3)^2+(0-4)^2+(1-5)^2}\ units=6units$.

Q25. A decorative item dealer deals in two items A and B. He has ₹15,000 to invest and a space to store at the most 80 pieces. Item A costs him ₹300 and item B costs him ₹150. He can sell items A and B at respective profits of ₹50 and ₹28. Assuming that he can sell all he buys, formulate the linear programming problem in order to maximize his profit and solve it graphically.

Sol. Let the dealer buys x and y number of item A and items B respectively.



To maximize, Z = ₹$(50x+28y)$

Subject to the constraints:

$300x+150y\le 15000$ i.e., $2x+y\le 100$ …(i)

$x+y\le 80$ …(ii)

and $x, y\ge 0$.

Considering the equations corresponding to the inequations (i) and (ii), we have:

$2x+y=100$

x	0	50
y	100	0

And, $x+y=80$

x	0	80
y	80	0

Take the testing points as (0, 0) for (i), we have: $2(0)+(0)\le 100 \Rightarrow 0\le 100$, which is true.

Take the testing points as (0, 0) for (ii), we have: $(0)+(0)\le 80 \Rightarrow 0\le 80$, which is true.

The shaded region as shown in the given figure is the feasible region, which is **bounded**.

The coordinates of the corner points of the feasible region are A(50, 0), B(0, 80), C(20, 60) and O(0, 0).
So, Value of Z at A(50, 0) = ₹2500, Value of Z at B(0, 80) = ₹2240, Value of Z at C(20, 60) = ₹2680, Value of Z at O(0, 0) = ₹0.

The maximum value of Z is ₹2680 which occurs at x = 20 and y = 60.

Hence, the dealer must buy 20 and 60 numbers of item A and items B respectively to maximize his profit. And the maximum profit obtained by him is ₹2680.

Q26. Evaluate $\int_2^5 (x^2+3)\,dx$ as limit of sums. **OR** Evaluate: $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x\,dx$.

Sol. As $\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$, *where* $nh=b-a$.

i.e., $\int_a^b f(x)dx = \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a+rh)$ …(i)

On comparing $\int_2^5 (x^2+3)\,dx$ with (i), we have: $f(x)=x^2+3,\ a=2,\ b=5$

So, $f(2+rh)=(2+rh)^2+3=7+4rh+r^2h^2$.

By using (i), $\int_2^5 (x^2+3)dx = \lim_{h\to 0} h\sum_{r=0}^{n-1}(7+4rh+r^2h^2)$

$\Rightarrow \quad = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1}7 + 4h\sum_{r=0}^{n-1} r + h^2\sum_{r=0}^{n-1} r^2\right] \quad \Rightarrow \quad = \lim_{h\to 0} h\left[7n + 4h\frac{n(n-1)}{2} + h^2\frac{n(n-1)(2n-1)}{6}\right]$

$\Rightarrow \quad = \lim_{h\to 0}\left[7nh + 2nh(nh-h) + \frac{nh(nh-h)(2nh-h)}{6}\right]$

Since when $n\to\infty,\ h\to 0$ and $nh=b-a=5-2=3$.

$\Rightarrow \quad = \left[7(3)+2(3)(3-0)+\frac{3(3-0)(6-0)}{6}\right] \quad \Rightarrow \quad = 9+18+21$

$\therefore \int_2^5 (x^2+3)\,dx = 48$.

OR Consider $\int \cos 2x \log \sin x\,dx$

$= \log\sin x\int \cos 2x\,dx - \int\left[\frac{d}{dx}(\log\sin x)\int \cos 2x\,dx\right]dx$ [Applying integral by parts

$= \left(\frac{\sin 2x}{2}\right)\log\sin x - \int\left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin 2x}{2}\right)dx$

$= \left(\frac{\sin 2x}{2}\right)\log\sin x - \int \cos^2 x\,dx \quad \Rightarrow \quad = \left(\frac{\sin 2x}{2}\right)\log\sin x - \int\left(\frac{1+\cos 2x}{2}\right)dx$

$= \left(\frac{\sin 2x}{2}\right)\log\sin x - \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)$

$= \frac{1}{2}\sin 2x\log\sin x - \frac{1}{4}\sin 2x - \frac{1}{2}x$

So, $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x \, dx = \left[\frac{1}{2}\sin 2x \log \sin x - \frac{1}{4}\sin 2x - \frac{1}{2}x\right]_{\pi/4}^{\pi/2}$

$\Rightarrow \quad = \left[\frac{1}{2}\sin \pi \log \sin \frac{\pi}{2} - \frac{1}{4}\sin \pi - \frac{1}{2}\left(\frac{\pi}{2}\right)\right] - \left[\frac{1}{2}\sin \frac{\pi}{2} \log \sin \frac{\pi}{4} - \frac{1}{4}\sin \frac{\pi}{2} - \frac{1}{2}\left(\frac{\pi}{4}\right)\right]$

$\Rightarrow \quad = -\frac{\pi}{4} - \left[\frac{1}{2}\log\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{4} - \frac{\pi}{8}\right] \qquad \Rightarrow \quad = -\frac{\pi}{4} - \left[-\frac{1}{4}\log 2 - \frac{1}{4} - \frac{\pi}{8}\right] = \frac{1}{4} + \frac{1}{4}\log 2 - \frac{\pi}{8}$

$\therefore \int_{\pi/4}^{\pi/2} \cos 2x \log \sin x \, dx = \frac{1}{4} + \frac{1}{4}\log 2 - \frac{\pi}{8}$.

Q27. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations given as: $x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1$.

Sol. Let $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$.

$\therefore \ AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$

$\Rightarrow \ = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \therefore \quad AB = 8I \qquad \ldots(i)$

Now the given system of equations are: $x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1$.

By using matrix method: let $C = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ and, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

Since $CX = D \qquad \Rightarrow X = C^{-1}D$

$\because \ B = C \Rightarrow C^{-1} = B^{-1}$ so, we have: $X = B^{-1}D \qquad \ldots(ii)$

By (i), we have: $AB = 8I \Rightarrow \left(\frac{1}{8}A\right)B = I \qquad \Rightarrow B^{-1} = \frac{1}{8}A \qquad \left[\because \ A^{-1}A = I = AA^{-1}\right.$

Using this in (ii), we get: $\qquad X = \frac{1}{8}AD$

$\Rightarrow X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} \Rightarrow \ = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} \quad \Rightarrow X = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

By equality of matrices, we have: $x = 3,\ y = -2,\ z = -1$.

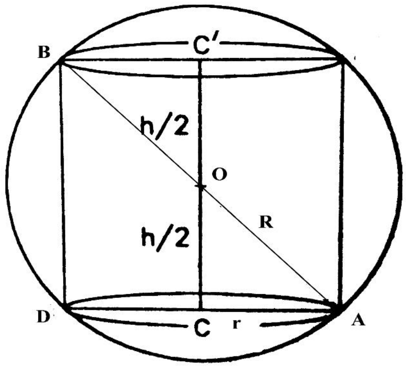
Hence, $x = 3,\ y = -2,\ z = -1$ is the required solution.

Q28. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$. Also find the maximum volume.

Sol. Let *r* and *h* be the radius and height of cylinder inscribed in a sphere of radius R.

In ΔBDA, we have: $\quad BD^2 + AD^2 = AB^2 \qquad$ [By using Pythagoras theorem

$\Rightarrow h^2 + (2r)^2 = (2R)^2 \qquad \Rightarrow r^2 = \frac{4R^2 - h^2}{4} \qquad \ldots(i)$



Now volume of the cylinder, $V = \pi r^2 h$

$$= \pi\left(\frac{4R^2 - h^2}{4}\right)h \qquad \text{[By using (i)}$$

$$\Rightarrow V = \frac{\pi}{4}\left(4R^2 h - h^3\right)$$

Differentiating w.r.t. h both the sides:

$$\frac{dV}{dh} = \frac{d}{dh}\left(\frac{\pi}{4}\left(4R^2 h - h^3\right)\right) \qquad \Rightarrow \frac{dV}{dh} = \frac{\pi}{4}\left(4R^2 - 3h^2\right)$$

Again differentiating w.r.t. h both the sides: $\frac{d^2V}{dh^2} = \frac{d}{dh}\left(\frac{\pi}{4}\left(4R^2 - 3h^2\right)\right) \Rightarrow \frac{d^2V}{dh^2} = -\frac{3\pi}{2}h.$

For points of local maxima & minima, $\frac{dV}{dh} = 0 \qquad \Rightarrow \frac{\pi}{4}\left(4R^2 - 3h^2\right) = 0$

$$\Rightarrow 3h^2 = 4R^2 \qquad \therefore \ h = 2R/\sqrt{3}.$$

Now, $\left.\frac{d^2V}{dh^2}\right]_{at\ h=\frac{2R}{\sqrt{3}}} = -\frac{3\pi}{2}\left(\frac{2R}{\sqrt{3}}\right) = -\sqrt{3}\,R\pi < 0.$ So, V is maximum at $h = \frac{2R}{\sqrt{3}}$.

Hence, height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$.

Also volume, $V = \frac{\pi}{4}\left[4R^2 - \frac{4R^2}{3}\right]\left(\frac{2R}{\sqrt{3}}\right) = \frac{4\pi R^3}{3\sqrt{3}}$ *cubic units*.

Hence maximum volume is $\frac{4\pi R^3}{3\sqrt{3}}$ *cubic units*.

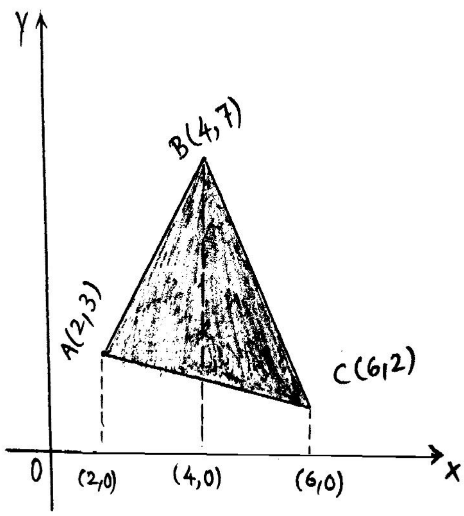
Q29. Using integration, find the area of the triangle ABC where A is (2, 3), B is (4, 7) and C is (6, 2).

Sol. We have A(2, 3), B(4, 7) and C(6, 2).

Equation of side AB: $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ 4 & 7 & 1 \end{vmatrix} = 0 \quad \Rightarrow -4x + 2y + 2 = 0 \quad \Rightarrow y = 2x - 1$ …(i)

Equation of side BC: $\begin{vmatrix} x & y & 1 \\ 4 & 7 & 1 \\ 6 & 2 & 1 \end{vmatrix} = 0 \quad \Rightarrow y = \frac{34 - 5x}{2}$ ...(ii)

Equation of side CA: $\begin{vmatrix} x & y & 1 \\ 6 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 0 \quad \Rightarrow y = \frac{14 - x}{4}$ ...(iii)



So, Required Area $= \int_2^4 y_{AB}dx + \int_4^6 y_{BC}dx - \int_2^6 y_{CA}dx$

$$= \int_2^4 (2x-1)dx + \frac{1}{2}\int_4^6 (34-5x)dx - \frac{1}{4}\int_2^6 (14-x)dx$$

$$= \frac{1}{4}\left[(2x-1)^2\right]_2^4 - \frac{1}{20}\left[(34-5x)^2\right]_4^6 + \frac{1}{8}\left[(14-x)^2\right]_2^6$$

$$= \frac{1}{4}\left[49 - 9\right] - \frac{1}{20}\left[16 - 196\right] + \frac{1}{8}\left[64 - 144\right]$$

i.e., *ar* (ABC) = 9 sq.units

Thus, area of the ΔABC is 9 square units.

Prepared by: **OP Gupta** ***[Electronics & Communications Engineering, Indira Award Winner]***
Contact me on: **+91-9650 350 480, theopgupta@gmail.com**